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Effects of the Initial Reynolds Number (Re_0) in the 3-Dimensional Shear Layer

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This paper considers the effects of initial Reynolds number in spatially developing viscous plane mixing layer. The large-scale structures are the embodiment of our desire to find order in apparent disorder. The objective of this paper is to investigate the role of three-dimensional large-scale coherent structures in mixing layer.

Key Words: Large-Scale Coherent Structures, Initial Reynolds Number, Spatial Problem, Reynolds Decomposition, Time and Spanwise Averaging

1. Introduction

This paper considers spatially developing, free shear layers at several Reynolds numbers, that are formed by the merging of two free streams initially separated by a splitter plate. A schematic diagram of the flow for a developing shear layer is sketched in Fig. 1. Shear layers are of practical importance in many fields where rapid transition to turbulence is desirable in order to prevent boundary layer separation or to promote rapid mixing. The ability to control mixing, structure and growth of shear flows have a considerable impact on many engineering applications. Free shear layers are also rich in fundamental mechanisms believed to be important in the transition process ti turbulence. One of the presently prevailing theories is that coherent structures originate as a result of instabilities of instantaneous disturbed flows. Thus from a fundamental point of view, one of the purposes of hydrodynamic stability theory is to understand the transition from laminar to turbulent flow. Coherent structures are the embodiment of our desire to find order in apparent disorder. It is well-organized large-scale component of the flow with phase-correlated vorticity(Hussain, 1986; Fiedler et al., 1991).

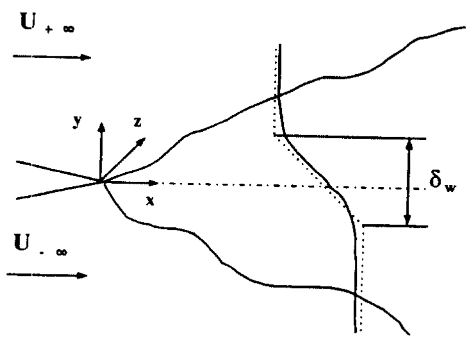

Fig. 1 Schematic diagram of a developing mixing layer

The objective of this study is to investigate the role of three-dimensional large-scale structures in the developing shear layer by the effects of the initial Reynolds numbers. To achieve this, we have to look into the mutual interactions among the three-dimentsional large-scale structure, the mean flow and purely two-dimensional large-scale structure.

2. Formulation of the Problem

The large-scale structures are going to be viewed as normal wave modes, products of hydrodynamic instability, which are periodic in

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space and time and grow with the streamwise coordinate alone (spatial problem). The flow will be decomposed into a mean component and five wave modes(Seo, 1993 ; Seo and Nikitopoulos, 1993). Several different terms such as large-scale coherent structure, wave mode, wave component, or orderly structure are used.

All flow variables can be decomposed into three components(Reynolds and Hussain, 1972) rather than the usual two as in the classical Reynolds decomposition. An arbitrary quantity $q(\vec{x}, t)$ is thus expressed as

$$q(\vec{x}, t)=Q(\vec{x})+\tilde{q}(\vec{x}, t)+q'(\vec{x}, t) \quad (1)$$

where $Q(\vec{x})$ is time-averaged mean quantity, $\tilde{q}(\vec{x}, t)$ represents the large-scale determistic component and $q'(\vec{x}, t)$ the small scale random component (fine grained turbulence). The interactions between the mean flow, two-dimensional large-scale coherent structure, and fine-grained turbulence have been previously studied by applying combined conditional and time-averaging techniques by Reynolds and Hussain(1972), Mankbadi and Liu(1981). Furthermore the experimental evidence(Weisbrot and Wygnanski, 1988 ; Ho and Huang, 1982) attest to the fact that the early development of the shear layer is dominated by the large-scale structures even if the shear layer is turbulent initially(Bell and Mehta, 1992) with high Reynolds number. However, since the present study is focused on the mutual interaction of multiple large-scale structure modes with the mean flow and with orther modes subjected to the initial paramenters, we will limit our analysis to the first two components of the decomposition as expressed by Eq. (1) and leave out the fine-grained turbulence component. Consequently we will employ a combined time and spanwise space average of our flow quantities expressed in general by

$$\bar{q}(\vec{x})=Q(\vec{x}) = \lim_{T\to\infty}\frac{1}{LT}\int_0^L\int_0^T q(\vec{x}, t)dtdz \quad (2)$$

where L is the spanwise wave-length, of the coherent strucuture and is equal to $\frac{2\pi}{\gamma}$ where γ is the spanwise wave number.

For the purpose of this study the large-scale velocity components $\tilde{u}_j$ and pressure $\tilde{p}$ are assumed to be Fourier analyzable, and are decomposed into five wave modes which are periodic in time t and the spanwise direction z. Thus the decomposition of the large-scale structure yields :

$$\begin{aligned}\tilde{u} &= \tilde{u}_{10}+\tilde{u}_{20}+\tilde{u}_{11}+\tilde{u}_{21}+\tilde{u}_{22}\\ \tilde{v} &= \tilde{v}_{10}+\tilde{v}_{20}+\tilde{v}_{11}+\tilde{v}_{21}+\tilde{v}_{22}\\ \tilde{p} &= \tilde{p}_{10}+\tilde{p}_{20}+\tilde{p}_{11}+\tilde{p}_{21}+\tilde{p}_{22}\\ \tilde{w} &= \tilde{w}_{11}+\tilde{w}_{21}+\tilde{w}_{22}.\end{aligned} \quad (3)$$

Each large-scale wave mode has the form :

$$\begin{bmatrix}\tilde{u}_{mn}\\ \tilde{v}_{mn}\\ \tilde{p}_{mn}\end{bmatrix}=|A_{mn}(x)|e^{i\psi_{mn}(x)}\begin{bmatrix}\hat{u}_{mn}\\ \hat{v}_{mn}\\ \hat{p}_{mn}\end{bmatrix}e^{-i\beta_{mn}t}\cos(n\gamma z) + c.c. \quad (4)$$

and

$$\tilde{w}_{mn}=|A_{mn}(x)|e^{i\psi_{mn}(x)}\ \hat{w}_{mn}e^{-i\beta_{mn}t}\sin(n\gamma z) + c.c. \quad (5)$$

where $\hat{u}_{mn}$, $\hat{v}_{mn}$, $\hat{p}_{mn}$ and $\hat{w}_{mn}$ are complex eigenfunctions to be calculated by the local linear stability analysis. β_{mn} is the frequency of the wave mode mn and $n\gamma$ is the wave number of the same. $A_{mn}(x)$ is complex amplitude in terms of its magnitude $|A_{mn}(x)|$ and phase angle $\psi_{mn}(x)$. $c.c.$ denotes the complex conjugate.

All fundamental modes (β_{20}, β_{21}, β_{22}) have the same frequency β_f and all subharmonic modes (β_{10}, β_{11}) have the frequency β_s*

$$\beta_f=2\beta_s. \quad (6)$$

We will also normalize the eigenfunction of each wave mode mn in a way that allow us to relate the amplitude $|A_{mn}(x)|$ to the corresponding average contents across the shear layer.

$$E_{mn}(x)=\frac{1}{2}\int_{-\infty}^{\infty}(\overline{\tilde{u}_{mn}^2}+\overline{\tilde{v}_{mn}^2}+\overline{\tilde{w}_{mn}^2})d\eta = |A_{mn}(x)|^2\delta(x) \quad (7)$$

where δ is the maximum slope thickness and η is the stretched cross-streamwise coordinate defined as $\frac{y}{\delta}$.

The following set of equations are obtained after some manipulation :

Mean flow ;

$$I_{aM}\frac{d\delta}{dx}=\frac{1}{Re_\delta}\Phi_M-\frac{1}{\delta}\sum_{mn}E_{mn}I_{MWmn}. \qquad (8)$$

Wave mode energies ;

$$I_{apWmn}\frac{dE_{mn}}{dx}=-\frac{1}{\delta}E_{mn}I_{MWmn}-\frac{1}{\delta Re_\delta}E_{mn}I_{vWmn}+EWW_{mn}. \qquad (9)$$

Wave mode phase angles ;

$$P_{aWmn}\frac{d\psi_{mn}}{dx}=\beta_{mn}+\frac{P_{WMmn}}{\delta}+\frac{1}{E_{mn}}\frac{dE_{mn}}{dx}P_{pWmn}+PWW_{mn}. \qquad (10)$$

The integral coefficients in the above system of equations are given by Seo(1993). In here we will show the most important integral coefficients. The wave mode production integral coefficient I_{MWmn} is

$$I_{MWmn}=\begin{cases}\int_{-\infty}^{\infty}2Real(\hat{u}_{mn}\ \hat{v}^*_{mn})\frac{\partial U}{\partial\eta}d\eta & \text{if } n=0,\\ \int_{-\infty}^{\infty}Real(\hat{u}_{mn}\ \hat{v}^*_{mn})\frac{\partial U}{\partial\eta}d\eta & \text{if } n\neq 0\end{cases} \qquad (11)$$

where Real represents the real part of the complex variable and ( )* denotes the complex conjugate. If $-I_{MWmn}$ is positive, then energy is transferred from the mean flow to the large-scale wave mode mn and if this term has opposite sign, the energy is transferred from the large-scale wave mode mn to the mean flow.

EWW_{mn} and PWW_{mn} can be represented as following :

$$EWW_{10}=\frac{1}{\delta^{\frac{3}{2}}}[E_{10}\sqrt{E_{20}}I^{10}_{1020}+\sqrt{E_{10}E_{11}E_{21}}(I^{21}_{1011}+I^{11}_{1021})]$$

$$EWW_{20}=\frac{1}{\delta^{\frac{3}{2}}}[E_{10}\sqrt{E_{20}}I^{10}_{1020}+E_{11}\sqrt{E_{20}}I^{11}_{1120}]$$

$$EWW_{11}=\frac{1}{\delta^{\frac{3}{2}}}[\sqrt{E_{10}E_{11}E_{21}}(I^{21}_{1011}-I^{10}_{1121})-E_{11}\sqrt{E_{20}}I^{11}_{1120}-E_{11}\sqrt{E_{22}}I^{11}_{1122}] \qquad (12)$$

$$EWW_{21}=\frac{1}{\delta^{\frac{3}{2}}}[\sqrt{E_{10}E_{11}E_{21}}(I^{11}_{1021}+I^{10}_{1121})]$$

$$EWW_{22}=\frac{1}{\delta^{\frac{3}{2}}}[E_{11}\sqrt{E_{22}}I^{11}_{1122}] \qquad (13)$$

$$PWW_{10}=\frac{1}{\delta^{\frac{3}{2}}}[\sqrt{E_{20}}P^{10}_{1020}+\frac{\sqrt{E_{11}E_{21}}}{\sqrt{E_{10}}}(P^{21}_{1011}+P^{11}_{1021})]$$

$$PWW_{20}=\frac{1}{\delta^{\frac{3}{2}}}[\frac{E_{10}}{\sqrt{E_{20}}}P^{10}_{1020}+\frac{E_{11}}{\sqrt{E_{20}}}P^{11}_{1120}]$$

$$PWW_{11}=\frac{1}{\delta^{\frac{3}{2}}}[\sqrt{E_{22}}P^{11}_{1122}+\sqrt{E_{20}}P^{11}_{1120}+\frac{\sqrt{E_{10}E_{21}}}{\sqrt{E_{11}}}(P^{10}_{1121}-P^{21}_{1011})] \qquad (14)$$

$$PWW_{21}=\frac{1}{\delta^{\frac{3}{2}}}[\frac{\sqrt{E_{10}E_{11}}}{\sqrt{E_{21}}}(P^{11}_{1021}+P^{10}_{1121})]$$

$$PWW_{22}=\frac{1}{\delta^{\frac{3}{2}}}[\frac{E_{11}}{\sqrt{E_{22}}}P^{11}_{1122}] \qquad (15)$$

where the wave mode-mode energy exchange terms, I^{pq}_{ijkl}, and the phase shift induced by interaction between wave modes, P^{pq}_{ijkl}, can be written as

$$I^{pq}_{ijkl}=2Real\Sigma^{pq}_{ij}\Delta_{kl}e^{-i((-)^p\psi_{pq}+(-)^i\psi_{ij}+(-)^k\psi_{kl})}$$
$$P^{pq}_{ijkl}=Im\Sigma^{pq}_{ij}\Delta_{kl}e^{-i((-)^p\psi_{pq}+(-)^i\psi_{ij}+(-)^k\psi_{kl})} \qquad (16)$$

The mode-mode interaction integral $\Sigma^{pq}_{ij}\Delta_{kl}$ can be presented in Appendix A.

From Eqs. (12)~(15) each subharmonic mode interacts directly with the two dimensional fundamental. The three dimensional subharmonic has the privilege of interacting with all fundamentals while the two dimensional subharmonic interacts with the fundamentals 20 and 21 wave modes. The subharmonics also communicate with each other. The fundamental 22 mode can only receive energy from the other modes.

3. Results and Discussion

The experimental studies by Konrad(1976), Breidenthal(1978) and Bernal and Roshko(1986) have indicated that certain aspects of the evolution of three dimensional shear layers depend on the Reynolds number. This gives a motivation for us to examine the effect of the Reynolds number on the evolution of the five mode interactions.

Results of initial Reynolds number variation are shown in Figs. (2)~(4) for equal strength of the modal energy densities, E_{mn}, for the two- and three-dimensional wave modes. The fundamental

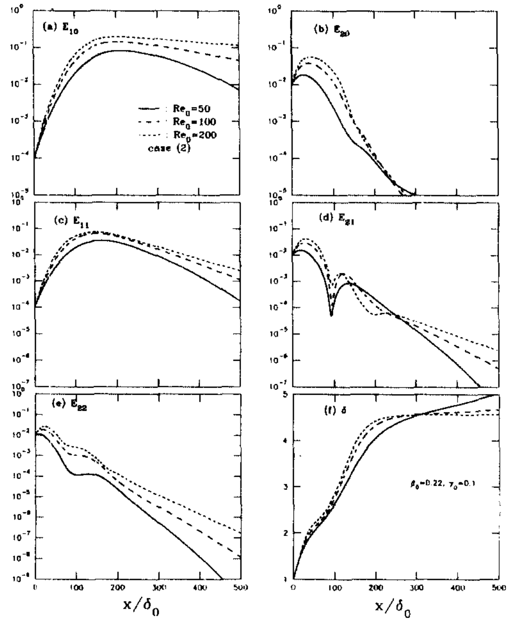

Fig. 2 Effect of the initial Reynolds number Re_0 for the development of modal energy densities E_{mn} and the growth of the shear layer δ

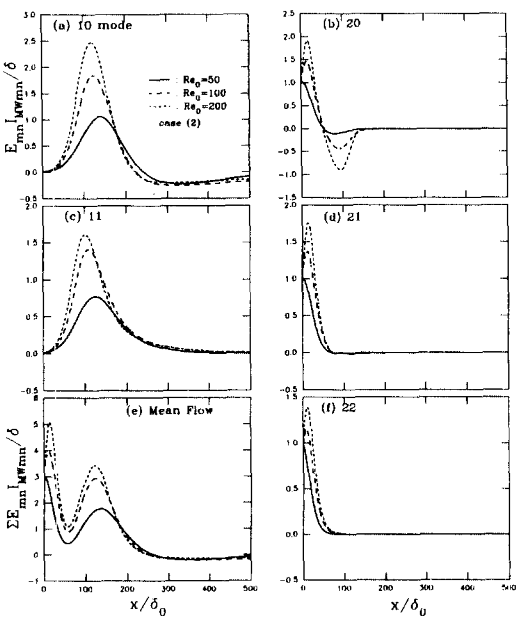

Fig. 3 Effect of the initial Reynolds number Re_0 for the energy interaction with the mean flow

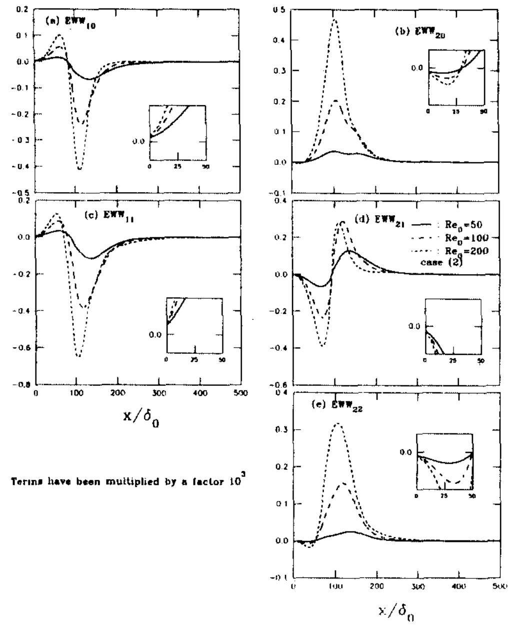

Fig. 4 Effect of the initial Reynolds number Re_0 for the energy interaction between modes

energies grow first, reach the maximum point where they become neutral, and start to decay because of loss of energy to the mean flow (see Figs. 2 and 3). The subharmonics also follow approximately the same course of the fundamentals but peak further downstream. As the subharmonics grow stronger, they extract an increasing amount of energies from the mean flow leading to the second peak in Fig. 2(f). Therefore the presence of the subharmonics are responsible for a significant increase in the growth of the shear layer and the subsequent increase in entrainment. The peaks of the fundamental wave mode energies are related with the first plateau of the shear layer thickness and the peaks of the subharmonic energies are related with the second plateau in the shear layer thickness further downstream. According to Ho and Huang(1982), and Huang(1985) the position of the peak of fundamentals is near the location of the first roll-up of shear layer and the vortex-merging is completed when the subharmonics have peaked. It is seen from Fig. 2 that the initial growth of all modes is reduced as the Reynolds number decreases. The reduced growth of the modes is primarily attributed to the reduced strength of the interaction of all of them with the mean flow (see Fig. 3). The peak of all the fundamentals increases monotonically with increasing initial Reynolds number and are also moved downstream. As a consequence, the growth of the shear layer at early stage of its development is more pronounced as the Reynolds number increases. The strength of the subharmonic peaks also increased with increasing Reynolds number partly because of the early favorable nonlinear wave mode interactions (see Fig. 4) and mainly because of the increased interaction with the mean flow (see Fig. 3).

According to Fig. 4 when the subharmonics are dominant far downstream, the energy transfer due to their nonlinear wave mode-mode interaction with fundamentals is almost unilaterally in favor of the latter. In the initial region the subharmonics gain energy from the fundamentals because of the nonlinear wave mode-mode interaction as shown in small windows in Fig. 4.

It is observed that as the initial Reynolds number increases the sensitivity of evolution of the shear layer and the wave modes are reduced. This is expected as the Reynolds number increases towards the inviscid limit. It is also observed that at lower Reynolds numbers shear layer sustains some noticeable growth even after the subharmonics have saturated and are decline. This is quite visible for the $Re_0=50$ case and is primarily due to the reduced ability of the subharmonics and the fundamentals (especially three-dimensional ones), to feed energy back to the mean flow and counter the effect of viscosity (see Fig. 3(a))

4. Conclusions

A theoretical investigation of the role of the three dimensional large-scale coherent structures and their mutual interactions in a developing plane mixing layers subjected to the initial Reynolds number is presented. Based on these results the following guidelines are presented :

(1) The initial growth of all modes is reduced as the Reynolds number decreases. The reduced growth of the modes is primarily attributed to the reduced strength of the interaction of all of them with the mean flow.

(2) The growth of the shear layer at the early stage of its development is more pronounced as the Reynolds number increases. The strength of the subharmonic peaks are also raised with increasing Reynolds number partly because of the early favorable nonlinear wave mode interactions.

Appendix A $\Sigma_{ij}^{pq}\Delta_{kl}$

$$\Sigma_{10}^{10}\Delta_{20}=\int_{-\infty}^{\infty}-i\alpha_{20}^{*}(\hat{u}_{10}^{2}\hat{u}_{20}^{*}-\hat{v}_{10}\hat{u}_{20}\hat{v}_{20}^{*})d\eta$$
$$+\int_{-\infty}^{\infty}(\hat{u}_{10}\hat{v}_{10}\frac{\partial\hat{u}_{20}^{*}}{\partial\eta}+\hat{v}_{20}^{2}\frac{\partial\hat{v}_{20}^{*}}{\partial\eta})d\eta$$

$$\Sigma_{11}^{11}\Delta_{20}=\int_{-\infty}^{\infty}-\frac{i\alpha_{20}^{*}}{2}(\hat{u}_{11}^{2}\hat{u}_{20}^{*}+\hat{v}_{11}\hat{u}_{11}\hat{v}_{20}^{*})d\eta$$
$$+\frac{1}{2}\int_{-\infty}^{\infty}(\hat{u}_{11}\hat{v}_{11}\frac{\partial\hat{u}_{20}^{*}}{\partial\eta}+\hat{v}_{11}^{2}\frac{\partial\hat{v}_{20}^{*}}{\partial\eta})d\eta$$

$$\Sigma_{10}^{21}\Delta_{11}=\frac{1}{2}\int_{-\infty}^{\infty}i\alpha_{11}(\hat{u}_{10}\hat{u}_{11}\hat{u}_{21}^{*}+\hat{v}_{10}\hat{v}_{11}\hat{u}_{21}^{*})d\eta$$
$$+\frac{1}{2}\int_{-\infty}^{\infty}(\hat{u}_{10}\hat{v}_{21}^{*}\frac{\partial\hat{u}_{11}}{\partial\eta}+\hat{v}_{10}\hat{v}_{21}^{*}\frac{\partial\hat{v}_{11}}{\partial\eta})d\eta$$
$$-\frac{1}{2}\int_{-\infty}^{\infty}\gamma(\hat{u}_{10}\hat{w}_{21}^{*}\hat{u}_{11}+\hat{v}_{10}\hat{w}_{21}^{*}\hat{v}_{11})d\eta$$

$$\Sigma_{10}^{11}\Delta_{21}=\frac{1}{2}\int_{-\infty}^{\infty}i\alpha_{21}(\hat{u}_{10}\hat{u}_{11}\hat{u}_{21}^{*}+\hat{v}_{10}\hat{v}_{11}\hat{u}_{21}^{*})d\eta$$
$$+\frac{1}{2}\int_{-\infty}^{\infty}(\hat{u}_{10}\hat{v}_{11}\frac{\partial\hat{u}_{21}^{*}}{\partial\eta}+\hat{v}_{10}\hat{v}_{11}\frac{\partial\hat{v}_{21}^{*}}{\partial\eta})d\eta$$
$$-\frac{1}{2}\int_{-\infty}^{\infty}\gamma(\hat{u}_{10}\hat{u}_{21}^{*}\hat{w}_{11}+\hat{v}_{10}\hat{v}_{21}^{*}\hat{w}_{11})d\eta$$

$$\Sigma_{11}^{10}\Delta_{21}=\int_{-\infty}^{\infty}-i\alpha_{21}^{*}(\hat{u}_{11}\hat{u}_{10}\hat{u}_{21}^{*}+\hat{v}_{11}\hat{v}_{10}\hat{u}_{21}^{*}$$
$$+\hat{w}_{11}\hat{u}_{10}\hat{w}_{21}^{*})d\eta$$
$$+\frac{1}{2}\int_{-\infty}^{\infty}(\hat{u}_{11}\hat{v}_{10}\frac{\partial\hat{u}_{21}^{*}}{\partial\eta}+\hat{v}_{11}\hat{v}_{10}\frac{\partial\hat{v}_{21}^{*}}{\partial\eta}$$
$$+\hat{w}_{11}\hat{v}_{10}\frac{\partial\hat{w}_{21}^{*}}{\partial\eta})d\eta$$

$$\Sigma_{11}^{11}\Delta_{22}=\frac{1}{4}\int_{-\infty}^{\infty}i\alpha_{22}^{*}(\hat{u}_{11}^{2}\hat{u}_{22}^{*}+\hat{v}_{11}\hat{u}_{11}\hat{v}_{22}^{*}+\hat{w}_{11}\hat{u}_{11}\hat{w}_{22}^{*})d\eta$$
$$+\frac{1}{4}\int_{-\infty}^{\infty}(\hat{u}_{11}\hat{v}_{11}\frac{\partial\hat{u}_{22}^{*}}{\partial\eta}+\hat{v}_{11}^{2}\frac{\partial\hat{v}_{22}^{*}}{\partial\eta}$$
$$+\hat{w}_{11}\hat{v}_{11}\frac{\partial\hat{w}_{22}^{*}}{\partial\eta})d\eta$$
$$-\frac{1}{2}\int_{-\infty}^{\infty}\gamma(\hat{u}_{11}\hat{w}_{11}\hat{u}_{22}^{*}+\hat{v}_{11}\hat{w}_{11}\hat{v}_{22}^{*}+\hat{w}_{11}^{2}\hat{w}_{22}^{*})d\eta$$

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